Constraints on plasma compensation of beam-beam effects in muon colliders

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We obtain necessary conditions for the plasma compensation to work in muon colliders. To this end, we analyze the suppression of beam fields by the plasma, collisional diffusion of the return plasma current, possible beam filamentation, and dynamics of plasma ions. We show that a good compensation requires very short beams and allows little freedom in choice of the plasma density.

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I. INTRODUCTION

A plasma can sustain extremely large electric fields. Due to this ability, various applications of plasmas to high-energy accelerators have been intensively studied [1–3]. Among them there are the wakefield acceleration, passive plasma lens, plasma guiding of beams, photon acceleration, and plasma compensation of beam fields at the interaction point of colliders. Here we consider the plasma compensation as applied to multi-TeV ultimate muon colliders [4].

Earlier the plasma compensation was first studied as a possible means of beamstrahlung suppression in linear electron-positron colliders [5,6], and it was realized that the plasma density required for good compensation is too high in future colliders; higher than the density of conduction electrons in solids. Later the suppression of beam-beam effects in circular colliders was considered [7], but degradation of the beam lifetime due to plasma turns out to be unacceptable for proton and electron machines. Here we derive necessary conditions for the plasma compensation to work in muon colliders and show that these conditions are difficult to satisfy. We analyze only the requirements imposed by the plasma compensation itself and leave aside the trade-off between the luminosity increase and the plasma-induced backgrounds since the latter will be considered elsewhere [8]. Also, we ignore the questions of multiturn beam stability as affected by the plasma.

II. SUPPRESSION OF THE BEAM FIELDS

The plasma can efficiently neutralize the electric and magnetic fields of the beam if the following conditions are fulfilled [5–7]:

$$n_e \gg n_b, \tag{1}$$

$$k_p \sigma_r \gg 1. \tag{2}$$

Here n_e is the electron density in the plasma, n_b is the beam density (the sum of beam densities in the case of several beams), σ_r is the rms beam radius, and k_p is the reciprocal plasma skin depth related to the plasma electron frequency ω_p as follows:

$$k_p = \frac{\omega_p}{c} = \sqrt{\frac{4\pi n_e e^2}{mc^2}}, \tag{3}$$

where m is the electron mass, c is the light velocity, and e is the elementary charge.

It is convenient to characterize the plasma compensation by the ratio of tune shifts with plasma (ξ) and without plasma (ξ_0). The smaller ξ/ξ_0 the better compensation. For round Gaussian beams, optimum plasma thickness, and idealized model of plasma behavior, this ratio is [9]

$$\frac{\xi}{\xi_0} \approx \frac{1}{k_p^2 \sigma_r^2} \left(1 + \frac{8 \ln(k_p^2 \sigma_r^2 - 1) + 1}{4\sqrt{\pi \ln(k_p^2 \sigma_r^2 - 1)}} \right). \tag{4}$$

Consequently, to achieve a given ratio ξ/ξ_0 we need, at least,

$$k_p \sigma_r \gtrsim \sqrt{a_s(\xi_0/\xi)}, \tag{5}$$

where

$$a_s \sim 1 + \frac{8 \ln(\xi_0/\xi) + 1}{4\sqrt{\pi \ln(\xi_0/\xi)}} \tag{6}$$

is a factor of the order of unity.

The plasma neutralizes the electric field of the beam much better than its magnetic field. It is uncompensated magnetic field that makes the dominant contribution to ξ. The electric field is v/c times smaller [9], where v is the longitudinal velocity of plasma electrons. However, the electric field always pushes plasma ions radially away from the beam axis [9].

Let us rewrite inequalities (1) and (5) in terms of number of particles in each colliding beam (N_b), beam size (σ_r, σ_z), plasma ion density (n_i), ion charge state ($Z = n_e/n_i$), and ''ion'' skin depth

$$\lambda\!\!\!^{-}_{pi} = \frac{c\sqrt{Z}}{\omega_p} = \sqrt{\frac{mc^2}{4\pi n_i e^2}}.$$

For Gaussian beams, we have the maximum beam density

$$n_b = \frac{N_b}{(2\pi)^{3/2} \sigma_z \sigma_r^2} \tag{7}$$

and

$$\frac{n_b}{n_e}=\sqrt{\frac{2}{\pi}}\frac{N_b r_e \lambda_{pi}^2}{Z\sigma_r^2\sigma_z}, \tag{8}$$

where r_e is the classical electron radius. Then Eq. (1) becomes

$$\sigma_z\sigma_r^2 \gg \sqrt{\frac{2}{\pi}}\frac{N_b}{Z} r_e \lambda_{pi}^2, \tag{9}$$

and Eq. (5) turns out to be

$$\sigma_r \gg \lambda_{pi}\sqrt{\frac{a_s(\xi_0/\xi)}{Z}}. \tag{10}$$

III. COLLISIONAL DIFFUSION OF THE RETURN CURRENT

When the beam (or two opposite beams in our case) enters the plasma, it inductively generates the plasma return current. This current provides an approximate local compensation and the exact integral compensation of the beam current. Due to electron-ion collisions in the plasma, the area of the return current broadens, and the local compensation becomes worse. This is the well-known phenomena of the diffusion of a magnetic field into a stationary conductor (see, e.g., Ref. [10]). Electron-electron collisions do not change the return current and affect the field diffusion only via heating of plasma electrons. Quantitatively, the magnetic diffusion is described by the equation

$$\frac{\partial B_c}{\partial t}=\frac{c^2}{4\pi\sigma}\Delta B_c, \tag{11}$$

where B_c is the azimuthal magnetic field of the return current, σ is the plasma conductivity, and Δ is the Laplacian. The plasma conductivity can be expressed in terms of electron-ion collision frequency ν_{ei} or electron velocity v:

$$\sigma=\frac{n_e e^2}{m\nu_{ei}}=\frac{mv^3}{4\pi\Lambda Z e^2}, \tag{12}$$

where Λ is the Coulomb logarithm.

To obtain the required compensation, the plasma has to reduce the magnetic field of the beam $\xi_0/(2\xi)$ times. The factor of 2 appears here because the electric field (that makes half the contribution to ξ_0) is always perfectly eliminated by the plasma. Then, as follows from Eq. (11), the beam should be short:

$$\frac{2\xi}{\xi_0} > \frac{1}{B_c}\frac{\partial B_c}{\partial t}\cdot\frac{\sigma_z}{c}=\frac{a_d\Lambda Z e^2 c\sigma_z}{mv^3\sigma_r^2}. \tag{13}$$

Here the constant $a_d \sim 1$ comprises possible errors introduced when we estimate the derivatives of B_c. Since the ‘‘number’’ of electron-ion collisions during the beam passage is

$$N_{\rm col}=\frac{\sigma_z\nu_{ei}}{c}<\frac{2k_p^2\sigma_r^2(\xi/\xi_0)}{a_d}\sim 1, \tag{14}$$

we can neglect the heating of plasma electrons [11] and put the velocity of plasma electrons equal to the drift velocity of the electron fluid:

$$v=b_d c n_b/n_e. \tag{15}$$

The factor $b_d \in (0,1)$ appears because at the beam periphery plasma electrons move slower than at the beam center. Substituting Eq. (15) into Eq. (13), we can rewrite the condition of admissible magnetic diffusion in the form of the limitation on beam dimensions

$$\sigma_r\sigma_z<\left(\frac{4\sqrt{2}(\xi/\xi_0)}{\pi\sqrt{\pi}a_d\Lambda}\right)^{1/4}\frac{(b_d N_b)^{3/4}}{Z} r_e^{1/2}\lambda_\pi^{3/2}. \tag{16}$$

We retain all numerical factors in formulas to avoid accumulation of errors.

IV. BEAM FILAMENTATION

Cold beams in the plasma are subject to filamentation. This phenomenon, known as a manifestation of Weibel instability, was studied in detail in application to plasma wakefield acceleration. It was found [12] that the beam is stable if the transverse component of its thermal velocity satisfies the condition

$$v_{b\perp}>c\sqrt{\frac{mn_b}{\gamma_b m_\mu n_e}}, \tag{17}$$

where γ and m_μ are the relativistic factor and rest mass of the beam particles, correspondingly. As follows from the derivation of Eq. (17), for two counterpropagating beams this formula is also valid at least by the order of magnitude.

Assume that the β function at the interaction point is equal to σ_z. Then

$$\frac{v_{b\perp}}{c}\sim\frac{\sigma_r}{\sigma_z}, \tag{18}$$

and the stability condition reads

$$\frac{\sigma_r^4}{\sigma_z}>\frac{a_f m N_b}{\gamma_b m_\mu Z} r_e \lambda_{pi}^2, \tag{19}$$

where $a_f \sim 1$ is a numerical factor.

V. MOTION OF PLASMA IONS

In the presence of an ultrarelativistic beam, a small radial electric field appears in the plasma. This field balances the magnetic force exerted on plasma electrons moving axially in the incompletely neutralized magnetic field of the beam [9]. The electric field always pushes plasma ions out of the beam region. When the ion density near the beam axis reduces to zero, any compensation of the beam fields (both electric and magnetic) disappears. Here we write out the limitation to beam parameters imposed by the dynamics of plasma ions.

The typical value of the radial electric field is

$$E \sim \frac{v}{c} B \sim \frac{v}{c} \cdot \frac{e n_b \sigma_r}{(k_p \sigma_r)^2} \sim \frac{a_i e n_b^2}{k_p^2 \sigma_r n_e}, \tag{20}$$

where the numerical factor $a_i \sim 1$ reflects an uncertainty in determination of E. This field shifts plasma ions radially by the distance $\sim \sigma_r$ in the time

$$\tau_i \sim \sqrt{\frac{M_i \sigma_r}{ZeE}}, \tag{21}$$

where M_i is the ion mass. For the compensation to take place we need

$$\sigma_z < b_i c \tau_i , \tag{22}$$

where $b_i \sim 1$. Substituting Eqs. (20), (21), and (8) into Eq. (22), we obtain

$$\sigma_r > \left(\frac{a_i m}{2\pi^2 b_i^2 Z M_i} \right)^{1/6} N_b^{1/3} r_e^{\,1/3} \bar{\lambda}_\pi^{2/3} . \tag{23}$$

Thus, for a good plasma compensation, the beam should be wide enough.

VI. COMBINED LIMITATIONS

Let us consider inequalities (9), (10), (16), (19), and (23) together and choose the most important ones. We take the following parameters as a reference point:

$$N_b = 5 \times 10^{12}, \quad \Gamma_b \equiv m_\mu \gamma_b / m \approx 10^7, \tag{24}$$

$$n_e = n_i \approx 5 \times 10^{22}\ \text{cm}^{-3}, \quad \bar{\lambda}_{pi} \approx 2.4 \times 10^{-6}\ \text{cm}, \tag{25}$$

$$M_i / m \approx 1.3 \times 10^4, \quad Z = 1, \quad \xi_0 / \xi = 10, \tag{26}$$

which correspond to a 5 TeV muon beam and conduction electrons of liquid lithium as the plasma. Then the above inequalities can be rewritten as

$$\sigma_z \sigma_r^2 / \bar{\lambda}_{pi}^3 \gg 4.7 \times 10^5, \tag{27}$$

$$\sigma_r / \bar{\lambda}_{pi} \gg 3.5, \tag{28}$$

$$\sigma_z \sigma_r / \bar{\lambda}_{pi}^2 < 3 \times 10^5, \tag{29}$$

$$\sigma_r^4 / (\sigma_z \bar{\lambda}_{pi}^3) > 0.06, \tag{30}$$

$$\sigma_r / \bar{\lambda}_{pi} > 10. \tag{31}$$

In derivation of Eqs. (27)–(31) we have put $\Lambda = 3$, $b_d = 0.5$, and $a_d = a_f = a_i = b_i = 1$. The areas determined by inequalities (27)–(31) are shown in logarithmic scale in Fig. 1(a). It is seen that, for the plasma compensation to work, the beam should be very short. The maximum beam length (point A in Fig. 1) can be found by substitution of the minimum σ_r [determined by Eq. (23)] into Eq. (16). It equals

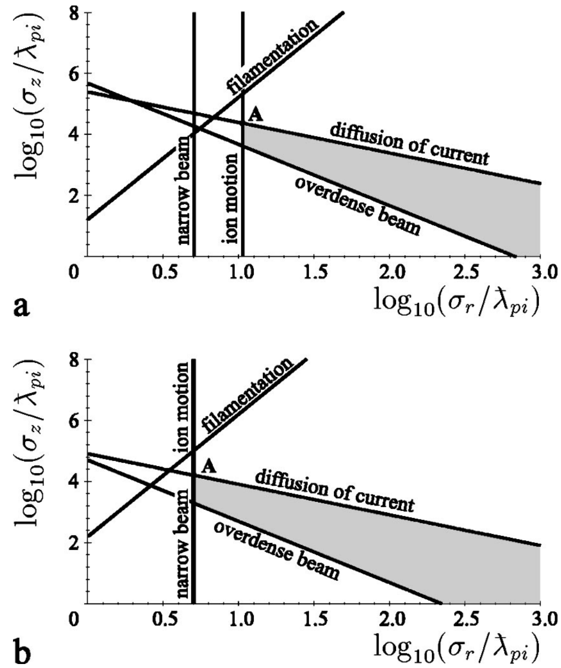

FIG. 1. Graphical representation of the limitations for plasma densities 5×10^{22} cm^{-3} (a) and 5×10^{20} cm^{-3} (b). In the shaded areas all the inequalities are fulfilled.

$$\sigma_{z,A} \approx \frac{6 b_i^{1/3} b_d^{3/4}}{a_i^{1/6} a_d^{1/4} \Lambda^{1/4}} \frac{M^{1/6} N_b^{5/12} r_e^{1/6} \bar{\lambda}_{pi}^{5/6}}{Z^{5/6} (\xi_0/\xi)^{1/4}}, \tag{32}$$

where M is the ion mass number.

We have $\sigma_{z,A} \approx 0.05$ cm for the above parameters. This value is much smaller than any conceivable bunch length. Using heavier metals as the plasma cannot save the situation because of the very weak dependence of $\sigma_{z,A}$ on M. Multiple ionization of the ions, which is possible at typical energies of plasma electrons, makes $\sigma_{z,A}$ appreciably shorter. More accurate calculations of the magnetic diffusion (a_d), electric field in the plasma (a_i), ion dynamics (b_i), or Coulomb logarithm (Λ) will not noticeably change the expression (32), because the dependence of $\sigma_{z,A}$ on the corresponding coefficients is weak. Possibly, a more accurate analysis of the plasma conductivity at different radii (b_d) can change the numerical factor in Eq. (32), but unlikely more than an order of magnitude.

As we see, the only way to make the plasma compensation work in muon colliders is to abandon the liquid metal plasma in favor of lower density plasmas. Inverting Eq. (32) and neglecting numerical factors of the order of unity,

$$\bar{\lambda}_{pi} \sim \frac{Z(\xi_0/\xi)^{3/10} \sigma_{z,A}^{6/5}}{2 M^{1/5} N_b^{1/2} r_e^{\,1/5}}, \tag{33}$$

we find that, for $\sigma_{z,A} = 0.3$ cm and all other parameters of Eqs. (24)–(26), the ion density should be

$$n_i \sim 5\times 10^{20}\ \mathrm{cm}^{-3} \quad (\lambda\!\!\!^{-}_{pi} \gtrsim 2\times 10^{-5}\ \mathrm{cm}). \tag{34}$$

The map of all limitations for this density is shown in Fig. 1(b). In comparison with Fig. 1(a) it gives an idea how the limitations change with variation of the plasma density.

For a fixed bunch length and a variable plasma ion density determined by Eq. (33), we deduce from Eq. (23) that

$$\sigma_r \sim \frac{0.1Z^{1/2}(\xi_0/\xi)^{1/5} r_e^{1/5} \sigma_{z,A}^{4/5}}{M^{3/10}}. \tag{35}$$

Thus, the longer the bunch, the wider it should be. Substituting $\sigma_{z,A}=0.3$ cm into Eq. (35), we obtain $\sigma_r \sim 1\ \mu$m. We see that the bunches longer than several millimeters are unacceptable since they are to be too wide. The decrease of the plasma density below the value of Eq. (34) is also unacceptable for this reason.

In the ultimate muon collider, the plasma compensation makes sense only if $\xi < \xi_{\max} \sim 0.1$. Let us determine the maximum beam length for which this requirement is fulfilled. For round beams,

$$\xi_0 = \frac{N_b r_e \sigma_z}{4\pi \Gamma_b \sigma_r^2}. \tag{36}$$

Substituting Eq. (35) into Eq. (36), putting $\xi=0.1$, and expressing σ_z in terms of ξ_0, we obtain

$$\sigma_z < \sigma_{z,\max} \sim \frac{7Mr_e}{\xi_0^{7/3}} \left(\frac{N_b}{\Gamma_b Z}\right)^{5/3} \sim \frac{0.04\ \mathrm{cm}}{\xi_0^{7/3}}. \tag{37}$$

For $\sigma_z = 0.3$ cm, we have $\xi_0 < 0.4$. Thus, for reasonably short bunches the plasma cannot save from too high tune shifts.

VII. CONCLUSION

We obtain several necessary conditions for the plasma compensation to work in colliders. To this end, we analyze the suppression of beam fields by the plasma, collisional diffusion of the return plasma current, possible beam filamentation, and dynamics of plasma ions.

For the ultimate muon collider, the most important (limiting) effects are the diffusion of the return current and the motion of plasma ions. To avoid the ion motion, the beams must be wide. To avoid the diffusion, the velocity of plasma electrons must be rather high, which requires a high density of the beams. With the beam radius fixed by the ion motion limit, the beam density has to be increased by decrease of the beam length.

For parameters of the ultimate 10 TeV muon collider, the required beam length is no longer than several millimeters. The beam length uniquely determines the required plasma density, that is of the order of $5\times 10^{20}\ \mathrm{cm}^{-3}$ in our case. However, for reasonably short bunches the plasma cannot save from too high tune shifts.

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